Mirror sym talk

Wednesday, July 1, 2020 3:47 PM

Plan

1. Big picture: HMS
2. Building the mirror
3. Wall crossing: fixing the mirror

(1) A side (symplectic)

(M,ω): monotone symplectic manifold

$$D^{\pi}\mathrm{Fuk}(M,\omega) = \bigoplus_{\lambda\in\mathbb{C}} D^{\pi}\mathrm{Fuk}_{\lambda}(M)$$

$D^{\pi}\mathrm{Fuk}_{\lambda}$ is trivial unless $\lambda \in \mathrm{spec}\left(QH(M) \xrightarrow{c_1 *} QH(M)\right)$

(due to Auroux–Kontsevich–Seidel)

B side (complex geometry)

(Y, W)

Y complex algebraic variety

$W: Y \longrightarrow \mathbb{C}$ holomorphic

$D^b\mathrm{Sing}(W^{-1}(w)) \to$ trivial unless w a critical value

$$D^{\pi}\mathrm{Fuk}_{w} \cong D^b\mathrm{Sing}(W^{-1}(w))$$

(See Sheridan HMS + Fano $\subset \mathbb{CP}^n$)

Q.: How to find (Y, W)?

Idea: (1) Take (M, ω, J) Kähler $\to$ remove D divisor

$M\setminus D$ is now Calabi Yau ($c_1(M\setminus D) = 0$)

Build mirror for $M\setminus D$

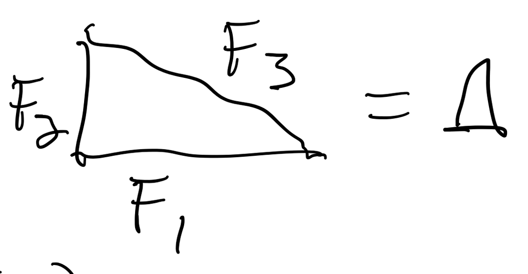
(2) SYZ mirror sym.

Log. torus fibration

$M\setminus D \longrightarrow B \qquad B = \{\text{special Lagrangians}\}$

dual fibration

Ex.: toric manifold

$M \xrightarrow{\mu}$  $= \Delta$

$D = \bigcup \mu^{-1}(F_i)$

$M\setminus D \longrightarrow \mathring{\Delta}$ (special) Log. torus fibration

Dual: replace the fiber (L) with

$L^{\vee} = \mathrm{Hom}(\pi_1(L); S^1) = \mathrm{Hom}(H_1(L); S^1)$

New base is $\{L \mid L = \mu^{-1}(p), p \in \mathring{\Delta}\}$

Locally, total space is

$X^{\vee} = U(L, L^{\vee}) \qquad X^{\vee} \longrightarrow \{L\}$

Fact: 1) $X^{\vee}$ admits $J^{\vee}, \omega^{\vee}, \Omega^{\vee}$ making $X^{\vee}$ Calabi-Yau.

2) $L^{\vee}$ is Lagrangian

Mirror for $M\setminus D$ $\to$ complete it to get mirror for M.

Conjecture: Mirror to M is $(X^{\vee}, W)$.

1) $X^{\vee}$ moduli of special Lags in $M\setminus D$ (completed and corrected)

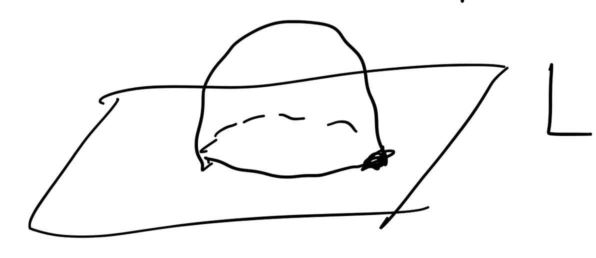
2) $W: X^{\vee} \longrightarrow \mathbb{C}$ is the Landau-Ginzburg potential, related Lag. Floer complex.

Superpotential

Nice case: L monotone Lagrangian

Fix $\beta \in H_2^{D}(M, L)$ then

$\mathcal{M}_{0,1}(\beta; J) \to J$ discs w. one marked point on ∂



Monotone: If $\mu(\beta) = 2$, then

$\mathcal{M}_{0,1}(\beta)$ is closed oriented manifold of $\dim = \dim L$

$ev: \mathcal{M}_{0,1}(\beta) \xrightarrow{ev} L$

$n_{\beta}(L) = \deg ev$

Facts: well defined

Fix $\varphi \in \mathrm{Hom}(H_1(L); S^1)$

Define $$m_0(L, \varphi) = \sum_{\substack{\beta \in H_2(M,L)\\ \mu(\beta)=2}} n_{\beta}(L)\, e^{-\omega(\beta)}\, \varphi(\partial\beta) \in \mathbb{C}$$

$m_0(L, \cdot)$ is holomorphic

Say $L = n$-torus $\quad H_1(L) = \mathbb{Z}^n = \langle e_i \rangle$

Pick z_i dual basis

$\partial\beta = \sum a_i \vec{e_i}$

$$m_0(L,\varphi) = m_0(L, (z_1, \dots, z_n)) = \sum_{\beta} n_{\beta}(L)\, e^{-\omega(\beta)}\, z_1^{a_1} z_2^{a_2} \cdots z_n^{a_n}$$

$\mathrm{dom}\; m_0(L, \cdot) = S^1 \times S^1 \times \cdots \times S^1$

L not always monotone... still:

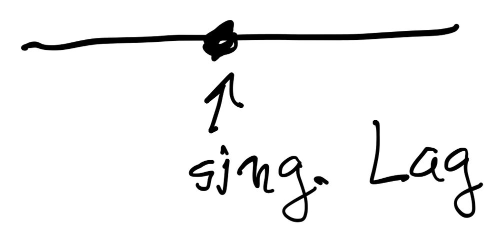
Facts when L is special

1) $\mu(\beta) = \partial^2 \cap D \geq 0$

2) In case $\dim L = 2$

most L_p satisfy $\mu \geq 2$

only isolated ones bound Maslov 0 discs



sing. Lag.